Control of the entanglement of a two-level atom in a dissipative cavity via a classical field

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Abstract

We investigate the entanglement dynamics and purity of a two-level atom, which is additionally driven by a classical field, interacting with a coherent field in a dissipative environment. It is shown that the amount of entanglement and the purity of the system can be improved by controlling the classical field.

Keywords: Cavity QED; Dissipation; Entanglement dynamics; Purity

PACS numbers: 03.67.Mn; 03.65.Ud

I. INTRODUCTION

Quantum entanglement is at the heart of quantum information processing and quantum computation [1]. It can exhibit a nonlocal correlation between quantum systems that can not be accounted for classically. The cavity QED is a useful tool to generate entangled states. It can be used to create entanglement between atoms in cavities and establish quantum communications between different optical cavities [2, 3, 4, 5, 6, 7, 8]. Many efforts have been devoted to the study of the manipulation of quantum entanglement with atoms and photons in cavities. However, a real quantum system is, in general, influenced by its surrounding environment[9]. The interaction between the quantum system and its environment leads to the so-called environment-induced decoherence. As a result, a pure quantum system may then become mixed and the amount of its entanglement will subsequently degrade.

In Ref.[10], Solano *et al.* have shown that multipartite entanglement can be generated by putting several two-level atoms in a cavity of high quality factor. In their paper, the Schrödinger cat state and other entangled states can be produced with the help of a strong classical driving fields. Generally, the entanglement of quantum states are fragile under the influence of decoherence. This is the most serious problem for all entanglement manipulation in quantum information processing. Up to now, various methods have been proposed to suppress decoherence, such as quantum error correction[11], decoherence-free subspaces[12], quantum feedback control[13], and dynamical decoupling[14].

In the present paper, we propose a scheme to enhance the amount of entanglement and purity of a quantum system consisting of a two-level atom interacting with coherent field in a dissipative environment by applying and controlling a classical driving field. We find an explicit expression of the density matrix of the system by making use of the superoperator algebraic approach and study the entanglement dynamics of the system by employing concurrence [15]. Our calculation shows that the amount of entanglement and the purity of the system can be enhanced by controlling the classical driving field.

II. THE MODEL

We consider a system consisting of a two-level atom interacting with a coherent field. The atom is driven by a classical field additionally. The Hamiltonian for the system can be described by [10]

$$\begin{aligned} H &= \omega a^\dagger a + \frac{\omega_0}{2}\sigma_z + g(\sigma_+ a + \sigma_- a^\dagger) \\ &\quad + \lambda(e^{-i\omega_c t}\sigma_+ + e^{i\omega_c t}\sigma_-), \end{aligned} \tag{1}$$

where ω, ω_0 and ω_c are the frequency of the cavity, atoms and classical field, respectively. The operators σ_z and $\sigma_\pm$ are defined by $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\sigma_+ = |e\rangle\langle g|$ where $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atom. Here, a and $a^\dagger$ are annihilation and creation operators of the cavity; g and λ are the coupling constants of the interactions of each atom with the cavity and with the classical driving field, respectively. Note that we have set $\hbar = 1$ throughout this paper.

In the rotating reference frame the Hamiltonian of the system is transformed to the Hamiltonian H_1 under a unitary transformation $U_1 = \exp(-i\omega_c t\sigma_z/2)$ [16]

$$H_1 = U_1^\dagger H U_1 - iU_1^\dagger \frac{\partial U_1}{\partial t} = H_1^{(1)} + H_1^{(2)}, \tag{2}$$

with

$$\begin{aligned} H_1^{(1)} &= \omega a^\dagger a + g(e^{i\omega_c t}\sigma_+ a + e^{-i\omega_c t}\sigma_- a^\dagger), \\ H_1^{(2)} &= \frac{\Delta_1}{2}\sigma_z + \lambda(\sigma_+ + \sigma_-), \end{aligned} \tag{3}$$

and $\Delta_1 = \omega_0 - \omega_c$. A straightforward calculation shows that the Hamiltonian $H_1^{(2)}$ can be diagonalized and recast as

$$H_1^{(2)} = \frac{\Omega_1}{2}\widetilde{\sigma}_z, \tag{4}$$

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where $\Delta_1 = \omega_0 - \omega_c$, $\Omega_1 = \sqrt{\Delta_1^2 + 4\lambda^2}$ and $\widetilde{\sigma}_z$ is defined by $\widetilde{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1|$. Here, $|0\rangle$ and $|1\rangle$ are dressed states

$$|0\rangle = \cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle, \quad |1\rangle = -\sin\frac{\theta}{2}|e\rangle + \cos\frac{\theta}{2}|g\rangle, \tag{5}$$

with $\theta = \arctan(\frac{2\lambda}{\Delta_1})$. Neglecting the terms which do not conserve energies (rotating wave approximation), we obtain the effective Hamiltonian H_1 in the dressed states

$$H_1 = \omega a^\dagger a + \frac{\Omega_1}{2}\widetilde{\sigma}_z + g\cos^2\frac{\theta}{2}(e^{i\omega_c t}\widetilde{\sigma}_+ a + e^{-i\omega_c t}\widetilde{\sigma}_- a^\dagger), \tag{6}$$

with $\widetilde{\sigma}_+ = |0\rangle\langle 1|$ and $\widetilde{\sigma}_- = |1\rangle\langle 0|$. The Hamiltonian (6) can be diagonalized by a final unitary transformation U_2 with $U_2 = \exp(\frac{i\omega_c t}{2}\widetilde{\sigma}_z)$. Using the identity $e^{-i\omega_c t\widetilde{\sigma}_z/2}\widetilde{\sigma}_+ e^{i\omega_c t\widetilde{\sigma}_z/2} = e^{-i\omega_c t}\widetilde{\sigma}_+$, we can rewrite the Hamiltonian of the system in the rotating reference frame

$$\begin{aligned} H_2 &= U_2^\dagger H U_2 - iU_2^\dagger\frac{\partial U_2}{\partial t} \\ &= \omega a^\dagger a + \frac{\omega'}{2}\widetilde{\sigma}_z + g'(\widetilde{\sigma}_+ a + \widetilde{\sigma}_- a^\dagger), \end{aligned} \tag{7}$$

where h.c stands for Hermitian conjugate, $\omega' = \Omega_1 + \omega_c = \sqrt{\Delta_1^2 + 4\lambda^2} + \omega_c$ and $g' = g\cos^2\frac{\theta}{2}$. It is worth noting that the unitary transformations U_1 and U_2 are both local unitary transformations. As we known the entanglement of a quantum system does not change under local unitary transformations. Thus, the entanglement of the system considered here will not change by applying local unitary transformations U_1 and U_2. Hereafter, unless specified otherwise we work in the rotating reference frame.

In the dispersive limit $|\Delta_2| = |\omega' - \omega| \gg \sqrt{n+1}g$, the interaction Hamiltonian $g'(a\widetilde{\sigma}_+ + a^\dagger\widetilde{\sigma}_-)$ can be regarded as a small perturbation. Using the method similar to that used in Ref.[16], we can recast the effective Hamiltonian (7) in the dispersive limit as

$$H_e = \omega a^\dagger a + \frac{\omega'}{2}\widetilde{\sigma}_z + \Omega[(a^\dagger a + 1)|0\rangle\langle 0| - a^\dagger a|1\rangle\langle 1|], \tag{8}$$

with $\Delta_2 = \omega' - \omega$ and $\Omega = \frac{(g\cos^2\frac{\theta}{2})^2}{\Delta_2}$.

III. SOLUTION

In this section, we investigate the entanglement dynamics of the two-level atom interacting with a coherent field in a dissipative environment by making use of the superoperator algebraic approach [17]. We assume that a classical driving field is applied additionally and the electromagnetic field couples to a reservoir. This interaction causes the losses in the cavity which is presented by the superoperator $\mathcal{D} = k(2a\cdot a^\dagger - a^\dagger a\cdot - \cdot a^\dagger a)$, where k is the decay constant. For the sake of simplicity, we confine our consideration in the case of zero temperature cavity. In the interaction picture, the interaction Hamiltonian is

$$V = \Omega[(a^\dagger a + 1)|1\rangle\langle 1| - a^\dagger a|1\rangle\langle 1|]. \tag{9}$$

Then, the master equation that governs the dynamics of the system can be written as follows

$$\begin{aligned} \frac{d\rho}{dt} &= -i[V,\rho] + \mathcal{D}\rho \\ &= -i[V,\rho] + k(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a). \end{aligned} \tag{10}$$

We can express the density matrix in the following form

$$\begin{aligned} \rho(t) &= \rho_{00}(t)\otimes|0\rangle\langle 0| + \rho_{11}(t)\otimes|1\rangle\langle 1| + \rho_{01}(t)\otimes|0\rangle\langle 1| \\ &\quad + \rho_{10}(t)\otimes|1\rangle\langle 0|, \end{aligned} \tag{11}$$

where ρ_{ij}'s are defined as $\rho_{ij} = \langle i|\rho|j\rangle$, $\rho_{ij} = \rho_{ji}^\dagger$, $i,j = 0,1$. A straightforward calculation shows that

$$\begin{aligned} \dot{\rho}_{00} &= \{-i\Omega(\mathcal{R} - \mathcal{L}) + k(2\mathcal{M} - \mathcal{R} - \mathcal{L})\}\rho_{00} = \mathcal{L}_{00}\rho_{00}(t), \\ \dot{\rho}_{11} &= \{i\Omega(\mathcal{R} - \mathcal{L}) + k(2\mathcal{M} - \mathcal{R} - \mathcal{L})\}\rho_{00} = \mathcal{L}_{11}\rho_{11}(t), \\ \dot{\rho}_{01} &= \{-i\Omega(\mathcal{R} + \mathcal{L} + 1) + k(2\mathcal{M} - \mathcal{R} - \mathcal{L})\}\rho_{00} \\ &= \mathcal{L}_{01}\rho_{00}(t), \\ \rho_{10} &= \rho_{01}^\dagger \end{aligned} \tag{12}$$

where $\mathcal{M}$, $\mathcal{R}$, and $\mathcal{L}$ are defined by

$$\mathcal{M} = a\cdot a^\dagger, \mathcal{R} = a^\dagger a\cdot, \mathcal{L} = \cdot a^\dagger a. \tag{13}$$

Here the superoperators $a\cdot, \cdot a, a^\dagger\cdot$ and $\cdot a^\dagger$ represent the action of creation and annihilation operators on an operator

$$(a\cdot)o = ao, (\cdot a)o = oa, \quad (a^\dagger\cdot)o = a^\dagger o, (\cdot a^\dagger)o = oa^\dagger. \tag{14}$$

It is easy to check that the superoperators $\mathcal{M}$, $\mathcal{R}$ and $\mathcal{L}$ satisfy the relations

$$[\mathcal{R},\mathcal{M}] = [\mathcal{L},\mathcal{M}] = -\mathcal{M}, [\mathcal{R},\mathcal{L}] = 0. \tag{15}$$

It is worth noting that $[\mathcal{R}+\mathcal{L},\mathcal{M}] = -2\mathcal{M}$, the superoperators $\mathcal{R}+\mathcal{L}$ and $\mathcal{M}$ form a shift operator algebra. Thus we have the expansion of the exponential of a linear combination of $\mathcal{R}+\mathcal{L}$ and $\mathcal{M}$

$$\begin{aligned} e^{\mathcal{L}_{00}t} &= e^{(e^{2kt}-1)\mathcal{M}}e^{-(i\Omega+k)t\mathcal{R}}e^{(i\Omega-k)t\mathcal{L}}, \\ e^{\mathcal{L}_{11}t} &= e^{(e^{2kt}-1)\mathcal{M}}e^{(i\Omega-k)t\mathcal{R}}e^{(-i\Omega+k)t\mathcal{L}}, \\ e^{\mathcal{L}_{01}t} &= e^{-i\Omega t}e^{(e^{2(i\Omega+k)t}-1)\mathcal{M}/(i\Omega+k)}e^{-(i\Omega+k)t\mathcal{R}}e^{-(i\Omega+k)t\mathcal{L}}. \end{aligned} \tag{16}$$

We assume the field is initially prepared in the coherent state $|\alpha\rangle$ and the atom is initially in state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Therefor the initial state of the atom-cavity system is $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes|\alpha\rangle$, i.e., $\rho_{00}(0) = \rho_{11}(0) = \rho_{01}(0) = \rho_{10}(0) = \frac{1}{2}|\alpha\rangle\langle\alpha|$. Combing Eq.(12) with Eq.(16), we find that the matrix elements $\rho_{ij}(t)$ at time t is given by

$$\begin{aligned} \rho_{00}(t) &= \frac{1}{2}|\alpha_+(t)\rangle\langle\alpha_+(t)|, \quad \rho_{11}(t) = \frac{1}{2}|\alpha_-(t)\rangle\langle\alpha_-(t)|, \\ \rho_{01}(t) &= \frac{1}{2}f(t)|\alpha_+(t)\rangle\langle\alpha_-(t)|, |\alpha_\pm(t)\rangle = |\alpha e^{-(k\pm i\Omega)t}\rangle, \\ f(t) &= \exp\{-i\Omega t + |\alpha|^2(e^{-2kt} - 1)\} \\ &\quad \times\exp\{\frac{|\alpha|^2 k}{k+i\Omega}(1 - e^{-2(k+i\Omega)t})\}. \end{aligned} \tag{17}$$

The density matrix of the atom-field system is then expressed as

$$\rho(t) = \frac{1}{2}\{|\alpha_+(t)\rangle\langle\alpha_+(t)| \otimes |0\rangle\langle 0| + |\alpha_-(t)\rangle\langle\alpha_-(t)| \otimes |1\rangle\langle 1| + [f(t)|\alpha_+(t)\rangle\langle\alpha_-(t)| \otimes |0\rangle\langle 1| + h.c]\}, \quad (18)$$

where h.c stands for Hermitian conjugate. In order to map the matrix onto a 2×2 system, we introduce two orthonormal vectors $|\uparrow\rangle$ and $|\downarrow\rangle$ which are defined by

$$|\uparrow\rangle = |\alpha_+(t)\rangle, |\downarrow\rangle = \frac{1}{\sqrt{1-|\tau|^2}}(|\alpha_-(t)\rangle - \tau|\alpha_+(t)\rangle), \quad (19)$$

with $\tau = \langle\alpha_+(t)|\alpha_-(t)\rangle$. Finally, the density matrix $\rho(t)$ now can be rewritten as

$$\rho(t) = \frac{1}{2}\{|\uparrow\rangle\langle\uparrow| \otimes |0\rangle\langle 0| + (\tau|\uparrow\rangle + \sqrt{1-|\tau|^2}|\downarrow\rangle) (\tau^*\langle\uparrow| + \sqrt{1-|\tau|^2}\langle\downarrow|) \otimes |1\rangle\langle 1| + [f(t)|\uparrow\rangle(\tau^*\langle\uparrow| + \sqrt{1-|\tau|^2}\langle\downarrow|) \otimes |0\rangle\langle 1| + h.c]\}, \quad (20)$$

where $h.c$ denotes for Hermitian conjugate.

IV. ENTANGLEMENT AND PURITY

In this section, we investigate the entanglement dynamics and purity of a quantum system consisting of a two-level atom, which is additionally driven by a classical field, interacting with a coherent field in a dissipative environment.

In order to study the entanglement of above system described by density matrix ρ, we adopt the measure concurrence which is defined by [15]

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (21)$$

where the λ_i(i=1,2,3,4) are the square roots of the eigenvalues in decreasing order of the magnitude of the "spin-flipped" density matrix operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ and σ_y is the Pauli Y matrix, i.e., $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Combing the definition of concurrence with the density matrix, we find that the concurrence of the system is

$$C(t) = |f(t)|\sqrt{1-|\tau|^2}. \quad (22)$$

In order to show the the effect of the classical field and the decay of the cavity on the entanglement dynamics of the system, we plot the the concurrence as a function of the coupling strength λ and the decay rate k in Fig.1. It is easy to see that the entanglement of the two-level atom and the cavity decreases with the increase of the decay rate k. However, the amount of entanglement between the two-level atom and the cavity can be increased by controlling the classical driving field as we can see from Fig.1.

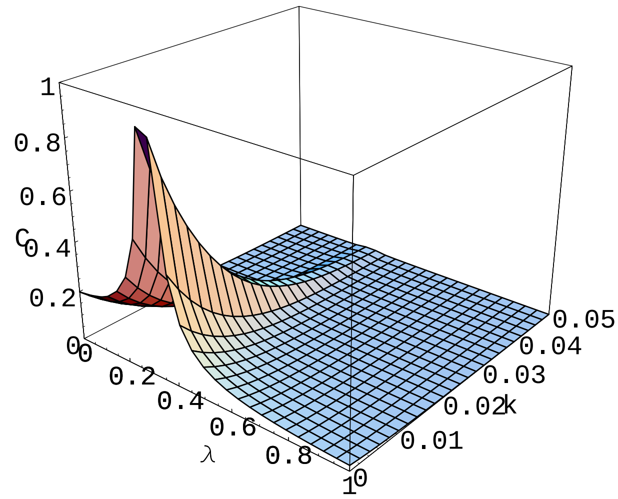

FIG. 1: The concurrence is plotted as a function of the coupling strength λ and the decay rate k with $\alpha = 1$, $g = 10^{-2}, t = 1/g, \omega = 2, \omega_0 = 1.9, \omega_c = 0$.

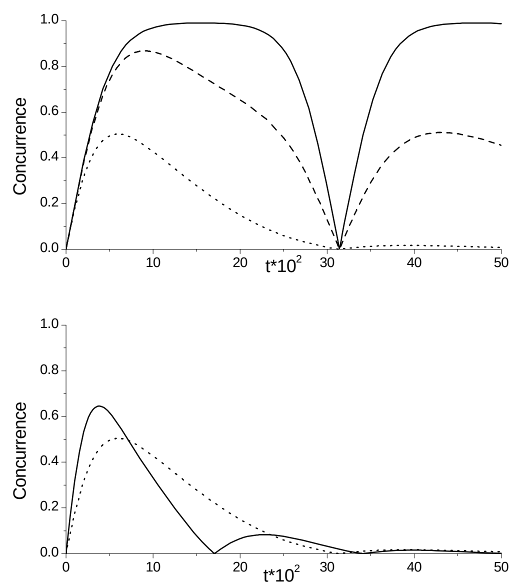

FIG. 2: Upper panel: The concurrence is plotted as a function of time with $\alpha = 1$, $g = 10^{-2}, \omega = 2, \omega_0 = 1.9, \omega_c = \lambda = 0$ for $k = 0$ (solid line), $k = 10^{-4}$ (dashed line), and $k = 10^{-3}$ (dotted line). Lower panel: The concurrence is plotted as a function of time with $\alpha = 1$, $g = 10^{-2}, \omega = 2, \omega_0 = 1.9, k = 10^{-3}$ for $\omega_c = \lambda = 0$ (dotted line) and $\omega_c = \lambda = 0.2$ (solid line).

In Fig.2, we plot the concurrence $C(t)$ as a function of time for different values k, ω_0, and λ. From the upper panel of Fig.2, one can see that the entanglement of the system decreases with the increase of the decay rate k. However, the maximal value of entanglement for the system can be improved by applying the classical driving field as one can easily find out in the lower panel of Fig.2.

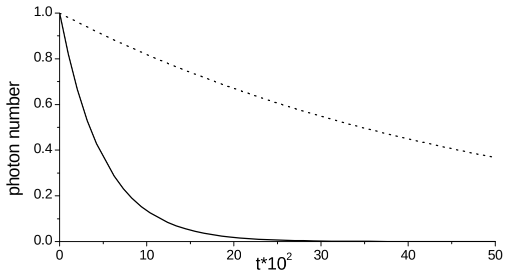

FIG. 3: The photon number of the cavity field is plotted as a function of time t with $\alpha = 1$ for $k = 10^{-4}$ (dotted line) and $k = 10^{-3}$ (solid line).

It is worth noting that in the case of $t \to \infty$ the states $|\alpha_{\pm}(t)\rangle = |\alpha e^{-(k \pm i\Omega)t}\rangle$ eventually goes to vacuum state $|0\rangle$. The photon number of the cavity field is

$$\langle n \rangle = Tr\{\rho(t) a^{\dagger} a\} = |\alpha|^2 e^{-2kt}. \tag{23}$$

It is easy to see that the photon number of the cavity field depends only on the photon number of the initial state $|\alpha|^2$, the decay rate k, and the time t. In Fig.3, we plot the photon number of the cavity field as a function of time t with $\alpha = 1$ for $k = 10^{-4}$ (dotted line) and $k = 10^{-3}$ (solid line). Comparing Fig.3 with Fig.4, we find that the photon number of the cavity field decreases with the increase of the time t while the concurrence is not a monotonic function of time t.

Next, we investigate the purity of the system by employing linear entropy. Many protocols in quantum information processing require pure, maximally entangled quantum states. For example, quantum teleportation often relies heavily on the purity and entanglement of the initial state. However, an pure and entangled quantum system usually becomes mixed and/or less entangled under the influence of decoherence. Here, we adopt the linear entropy to quantify the mixedness of a state defined by $S(\rho) = 1 - Tr(\rho^2)$. Generally, if ρ is the density matrix of a pure state, $S = 0$, otherwise $S > 0$. It has also been proved that a bipartite mixed states is useless for quantum teleportation if its linear entropy exceeds 1/2 for a two qubits system. The purity of the atom-field system is

$$S(\rho) = 1 - Tr(\rho^2) = \frac{1}{2}[1 - |f(t)|^2]. \tag{24}$$

In Fig.4, the linear entropy $S(\rho)$ is plotted as a function of time with (solid line) or without (dotted line) the classical driving field. As one can see clearly from Fig.4, the classical driving field can decrease the linear entropy of the atom-field system. In other words, the purity of the atom-field system can be significantly increased by applying the classical driving field.

V. CONCLUSION

In the present paper, we propose a scheme to improve the amount of entanglement and purity of a quantum system consisting of a two-level atom interacting with a coherent field in a dissipative cavity by applying and controlling a classical driving field. We find an explicit expression of the density matrix of the system and study the entanglement dynamics of the system by employing concurrence. Our calculation shows that the amount of entanglement and the purity of the system can be enhanced by applying the classical driving field. The approach adopted here can be extended to the system formed by two or more two-level atoms in dissipative cavities.

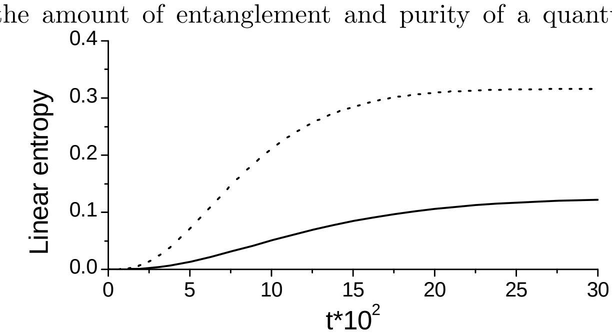

FIG. 4: The concurrence is plotted as a function of time with $\alpha = 1,\ g = 10^{-2}, \omega = 2, \omega_0 = 1.9, k = 10^{-3}$ for $\omega_c = \lambda = 0$ (dotted line) and $\omega_c = \lambda = 0.5$ (solid line).

Acknowledgments

This project was supported by the National Natural Science Foundation of China (Grant no.10774131) and the National Key Project for Fundamental Research of China (Grant no. 2006CB921403).

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